Math 207: Diff. Eq. - Dr. Loveless

Essential Course Info

Canvas Site: canvas.uw.edu/courses/1548377

Material Site: math.washington.edu/~aloveles/

Homework Log-In (use UWNetID): webassign.net/washington/login.html

First week to do list

1. Complete the Welcome Survey on Canvas (by Friday)
2. Read 1.1-1.3, 2.1-2.2 of the book. Start attempting HW 1 (which closes next Tues)
3. Be ready for short (10-minute) entry task on Friday
   - one integral (like one of these: m307BasicIntegrationExamples.pdf)
   - one short diff. eq. set up (like one of these: DifferentialEquationApplications.pdf)

Today

- Syllabus/Intro
- Section 1.1
  - diff. eq. examples
  - checking a solution

HW 1 Notes

Problems 1-3: Skills review

Problems 4-6: Checking solutions

Problems 7-9: Application Intro

Problems 10-15: Slope Fields

What we will do in this course:
Learn terminology, examples, and solving methods for 1st and 2nd order ordinary differential eqns.

1. **Ch. 1 – Intro/Applications**
   - Applications
   - Slope/direction fields

2. **Ch. 2 – First Order ODE's**
   - Separable equations
   - Linear eqs (integrating factor)
   - Equilibrium solns
   - Euler's approximation method

3. **Ch. 3 – Second Order ODE's**
   - Characteristic equations
   - Undetermined coefficients
   - Applications to oscillators

4. **Ch. 6 – LaPlace Transforms**
   - Important systematic method for solving differential equations

Chapter 1: Intro to Diff. Equations

Some terminology

- A *differential equation* is any equation involving a derivative.

- An *ordinary differential equation (ODE)* is an equation involving derivatives relating only two variables (dependent/independent).

- A *partial differential equation (PDE)* is an equation involving partial derivatives.

We only study ODE's in this course.

- The *order* of an ODE is the highest derivative that appears in the equation.

Checking Solutions *(like 4-6 in HW)*

An *explicit solution* to a differential equation is a function that satisfies the equation.

Example 1: $y' - 4y = 0$ has one solution that look like $y(t) = e^{rt}$. Find *r*.

Example 2: $y'' - \frac{6}{t^2}y = 0$ has two solutions that look like $y(t) = t^r$. Find the two values of *r*.

Example 3: $y'' - y' = 3x$ has one solution that look like

$y(t) = e^x + rx + sx^2$. Find *r* and s.

Applications/Units (Prob. 7 in HW)

$$\frac{dy}{dt} =$$

"rate of change of y with respect to t"

Example:

Let $P(t)$ ="population after t years".
Assume the population grows at a rate proportional to its size. That is,

$$\frac{dP}{dt} = kP$$

Units Discussion:

What if P is in thousands of people?

Such as $P_2(t) = \frac{P(t)}{1000}$ thousand people.

What is $\frac{dP_2}{dt}$?

What if t is in days instead of years?

Such as $t_2 = 365t$ days.

What is $\frac{dP}{dt_2}$?

Some Motivation

First, see handout on applications.

Please understand that we will discuss application throughout the first three weeks and then in section 2.3 we will go into them in more detail (i.e. there is more to come, today is just a taste).

Newton's Law of Cooling (HW Prob. 8)

"The rate of temperature change for an object is proportional to the difference between the temperature of the object and its surroundings."

$$\frac{dT}{dt} = k(T_s - T)$$

Examples involving motion

Recall: Newton's Law

$$ma = F \quad (Force)$$

If we write $a(t) = v'(t) = h''(t)$, then we see this leads to an ODE.

Air Resistance:

Assume an object with mass m is dropped from 1000 meters with an initial velocity of $v(0) = 0$ m/s.

From Newton's law, if we know the forces then we can set up a differential equation describing the motion! The question is what forces do we want to take into account…

Model 1:

Assuming no air resistance, then the only force is gravity F = -mg (g = 9.8 m/s²).

$$mv' = F_g = -mg$$

I*nitial* ***V****alue* ***P****roblem* (IVP)

$$v' = -g$$
$$v(0) = 0$$

Model 2:

Assume air resistance exerts a force proportional to speed in the opposite direction of motion.

$$mv' = F_g + F_A = -mg - rv$$

I*nitial* ***V****alue* ***P****roblem* (IVP)

$$v' = -g - \frac{r}{m}v$$
$$v(0) = 0$$

Side-note: I set this up assuming upward velocity is positive and downward velocity is negative. Sometimes textbooks like to set it up so that downward velocity is positive instead which leads to the equations:

$v' = +g$ *for model 1, and* $v' = +g - \frac{r}{m}v$ *for model 2.*

Morale: You want to be clear about your conventions and labeling before setting up a problem.

Mass-Spring Example:

Assume an object with mass m is attached to a spring that is attached to a wall. Natural length is the distance from the wall at which the mass is at rest (no stretch or push force).

Let x = “the distance the spring is stretched beyond natural length”.

Hooke’s Law says force due to the spring is proportional to x and in the opposite direction. In other words,

$$Force = -kx$$

So, once again, if you want to model the motion of the mass after you stretch it and let go, you use Newton's Law:

$$ma = F = -kx$$

and $a(t) = x''(t)$

So

$$m\,x'' = -kx$$

It turns out that one solution to this is $x(t) = cos(\omega t)$

Aside: This is called a "simple harmonic oscillator" and ω is the "natural frequency" (radians/time). And $2\pi/\omega$ is the wavelength (time from peak-to-peak)

We will study mass-springs a lot in chapter 3.

A random other example

Circuits Example:

Kirchoff's laws observe that the sum of the voltage drops in a circuit equals zero (source, resistance, capacitance, inductance).

So $-V_0 + V_R + V_C + V_L = 0$

with

$$I = \frac{dq}{dt} = q'$$

$$V_R = RI = Rq'$$

$$V_C = \frac{q(t)}{C} = \frac{q}{C}$$

$$V_L = L\frac{dI}{dt} = Lq''$$

So $V_0 = V_R + V_C + V_L$

$$V_0 = Rq' + \frac{q}{C} + Lq''$$

And you have a second order differential equation!

We see over and over that assumptions about rates, lead to differential equations. Hope this gives some motivation to you!

Okay, I hope that gives you a taste of several examples.

In the next class we will stop talking about applications and talk about slope fields and how to start analyzing differential questions without explicitly solve.

Then in the days to follow we will learn a couple explicit solving techniques.

So that is the plan, let’s have a good first week!